Toward an improved simulation of ocean-atmosphere interactions

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Joint work with F. Lemarié, C. Pelletier and S. Thery (LJK Grenoble)
in collaboration with many colleagues from the ANR COCOA project

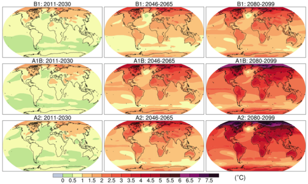
LABORATOIRE JEAN KUNTZMANN
MATHÉMATIQUES APPLIQUÉES - INFORMATIQUE

UNIVERSITÉ Grenoble Alpes

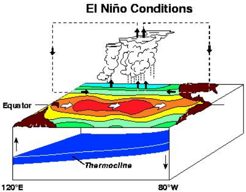
informatics mathematics

Context

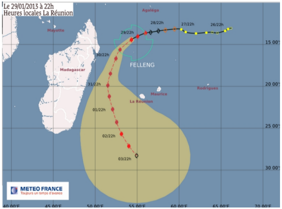
- Various applications require coupling an oceanic model and an atmospheric model.



climate modeling



seasonal forecasts

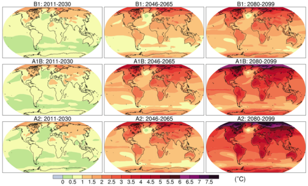


short term predictions

- complex interactions
- physical / mathematical / numerical / algorithmic issues

Context

- Various applications require coupling an oceanic model and an atmospheric model.



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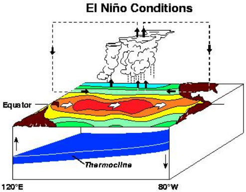

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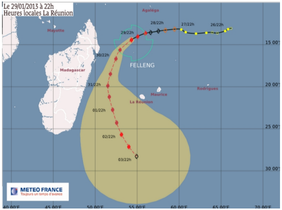


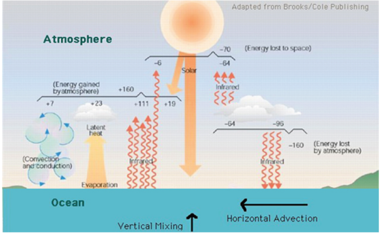
short term predictions

Objectives

- understand and criticize air-sea interactions modeling
- revisit the coupling strategies presently used in such systems

Air-sea interactions

$$\begin{cases} \mathcal{L}_{\mathrm{atm}}\, \boldsymbol{U}^{\mathrm{a}} = f_{\mathrm{atm}} & \text{in } \Omega_{\mathrm{atm}} \times [0, T] \\ \mathcal{L}_{\mathrm{oce}}\, \boldsymbol{U}^{\mathrm{o}} = f_{\mathrm{oce}} & \text{in } \Omega_{\mathrm{oce}} \times [0, T] \end{cases}$$



with $\boldsymbol{U}^{\mathrm{a}} = \begin{pmatrix} \boldsymbol{u}_h^{\mathrm{a}} \\ T^{\mathrm{a}} \end{pmatrix} \quad \boldsymbol{U}^{\mathrm{o}} = \begin{pmatrix} \boldsymbol{u}_h^{\mathrm{o}} \\ T^{\mathrm{o}} \end{pmatrix}$

Interface conditions:

momentum $\quad \rho^{\mathrm{a}} K_{\mathrm{m}}^{\mathrm{a}} \dfrac{\partial \boldsymbol{u}_h^{\mathrm{a}}}{\partial z} = \rho^{\mathrm{o}} K_{\mathrm{m}}^{\mathrm{o}} \dfrac{\partial \boldsymbol{u}_h^{\mathrm{o}}}{\partial z} = \boldsymbol{\tau} \quad$ on $\Gamma \times [0, T]$

heat flux $\quad \rho^{\mathrm{a}} c_p^{\mathrm{a}} K_{\mathrm{T}}^{\mathrm{a}} \dfrac{\partial T^{\mathrm{a}}}{\partial z} = \rho^{\mathrm{o}} c_p^{\mathrm{o}} K_{\mathrm{T}}^{\mathrm{o}} \dfrac{\partial T^{\mathrm{o}}}{\partial z} = Q_S + \mathcal{R} \quad$ on $\Gamma \times [0, T]$

Air-sea interactions

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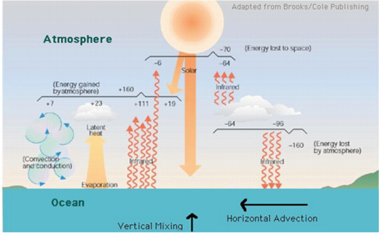

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Estimation of OA fluxes: modeling of surface boundary layer

1D primitive equations for the ocean and the atmosphere

Navier-Stokes in a rotating frame

+ standard approximations (Boussinesq, hydrostaticity, horizontal homogeneity)

+ **Reynolds average** $X = < X > + X'$

In Ω_α, $\alpha \in \{o, a\}$:

$$\begin{aligned}
\partial_t u - f v + \partial_z \langle w' u' \rangle &= 0 \\
\partial_t v + f u + \partial_z \langle w' v' \rangle &= 0 \\
\partial_z w &= 0 \\
\partial_z \widetilde{p} + \tilde{\rho} g &= 0 \\
\tilde{\rho} + \rho_{0,\alpha} &= \rho(\theta, S, p, q) \\
\partial_t \theta + \partial_z \langle w' \theta' \rangle &= \mathcal{F}_\theta \\
\partial_t S + \partial_z \langle w' S' \rangle &= 0 \quad \alpha = o \\
\partial_t q + \partial_z \langle w' q' \rangle &= 0 \quad \alpha = a
\end{aligned}$$

unknowns

constants

turbulent scales

turbulent boundary layer

$$\underbrace{\nu_\alpha^m}_{\text{molecular}} \ll \underbrace{\nu_\alpha^t}_{\text{turbulent}}$$

1D primitive equations for the ocean and the atmosphere

Navier-Stokes in a rotating frame
+ standard approximations (Boussinesq, hydrostaticity, horizontal homogeneity)
+ **turbulent closure**

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unknowns
constants
parameterizations

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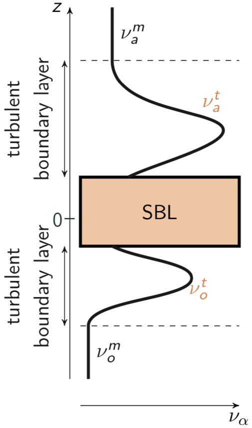


Boundary conditions: turbulent parameterizations

Via **frictional scales** $\mathbf{x}_a^* = (u_a^*, \theta_a^*, q_a^*)$ et $\mathbf{x}_o^* = (u_o^*, \theta_o^*, s_o^*)$

1) Artificial viscosities and diffusivities

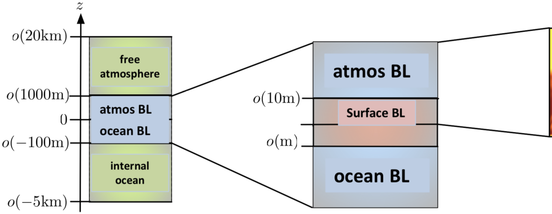
$$\begin{aligned} \nu_\alpha^t &= \nu_\alpha^t(\mathbf{x}_\alpha^*, z) \\ \mathcal{K}_x^t &= \mathcal{K}_{\alpha,x}^t(\mathbf{x}_\alpha^*, z) \end{aligned}$$

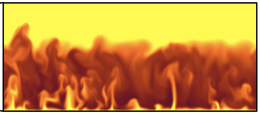
2) Turbulent boundary conditions

At $z = z_\alpha^1$:

$$\begin{aligned} \nu_\alpha^t \partial_z \mathbf{u_h} &= \boldsymbol{\tau}/\rho_{0,\alpha} = (u_\alpha^*)^2 \, \mathbf{e}_\tau \\ \mathcal{K}_{\alpha,\theta}^t \partial_z \theta &= u_\alpha^* \theta_\alpha^* + \mathcal{F}_{\theta,ext}^\alpha \\ \mathcal{K}_{a,q}^t \partial_z q &= u_a^* q_a^* + \mathcal{F}_{q,ext}^a \\ \mathcal{K}_{\alpha,S}^t \partial_z S &= u_o^* s_o^* + \mathcal{F}_{q,ext}^o \end{aligned}$$

$\mathbf{u_h} = (u, v)$ and $\mathcal{F}_{x,ext}^\alpha$ known ext. flux





(Conzemius & Fedorovich, 2008)

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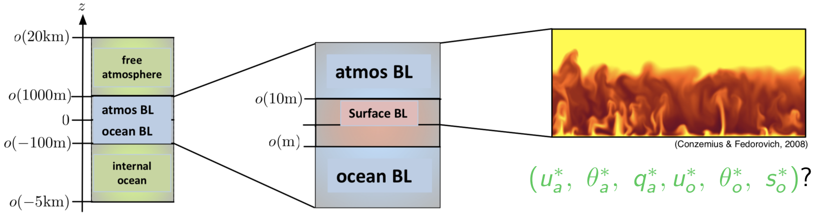

(Conzemius & Fedorovich, 2008)

$(u_a^*,\ \theta_a^*,\ q_a^*, u_o^*,\ \theta_o^*,\ s_o^*)$?

Closure equations on $(u_a^*,\ \theta_a^*,\ q_a^*,)$

Monin-Obukhov similarity theory (1954)

$$
\begin{aligned}
\left\| [\![\mathbf{u_h}]\!]_{z_u^{a,r}}^{z_a^1} \right\| &= \frac{u_a^*}{\kappa}\left[\ln\left(\frac{z_a^1}{z_u^{a,r}}\right) - \psi_m\left(\frac{z_a^1}{L_O^a(\mathbf{x}_a^*)}\right) + \psi_m\left(\frac{z_u^{a,r}}{L_O^a}\right)\right] \\
[\![\theta]\!]_{z_\theta^{a,r}}^{z_a^1} &= \frac{\theta_a^*}{\kappa}\left[\ln\left(\frac{z_a^1}{z_\theta^{a,r}}\right) - \psi_h\left(\frac{z_a^1}{L_O^a(\mathbf{x}_a^*)}\right) + \psi_h\left(\frac{z_\theta^{a,r}}{L_O^a}\right)\right] \\
[\![q]\!]_{z_q^{a,r}}^{z_a^1} &= \frac{q_a^*}{\kappa}\left[\ln\left(\frac{z_a^1}{z_q^{a,r}}\right) - \psi_h\left(\frac{z_a^1}{L_O^a(\mathbf{x}_a^*)}\right) + \psi_h\left(\frac{z_q^{a,r}}{L_O^a}\right)\right]
\end{aligned}
$$

$[\![X]\!]_{z_1}^{z_2} = X(z_2) - X(z_1)$, κ von Kármán constant, $\left(z_x^{a,r}\right)$ roughness lengths, L_O^a Obukhov length, ψ_x stability fns

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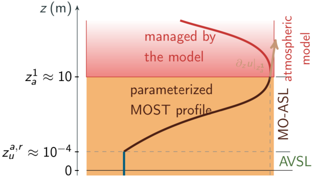
Two (universal but hidden) approximations:

- Neglect the stratification in the viscous sublayers $]0;\ z_x^{a,r}[$
- $X\left(z_x^{a,r}\right) \approx X\,(z=0)$ for the jumps



Roughness lengths

Parameterizations: $z_x^{a,r} = z_x^{a,r}\left(u_a^*, \theta_a^*, q_a^*\right)$

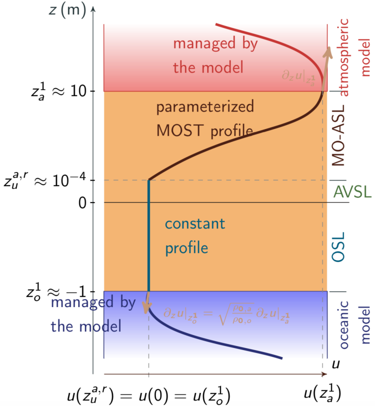
$\longrightarrow$ nonlinear system of equations for $\mathbf{x}_a^* = \left(u_a^*, \theta_a^*, q_a^*\right)$ "bulk formulas"

$$
\begin{aligned}
\frac{\kappa \left\| [\![\mathbf{u_h}]\!]_0^{z_a^1} \right\|}{u_a^*} &= \ln\left(\frac{z_a^1}{z_u^{a,r}\left(\mathbf{x}_a^*\right)}\right) - \psi_m\left(\frac{z_a^1}{L_O^a\left(\mathbf{x}_a^*\right)}\right) \\
\frac{\kappa [\![\theta]\!]_0^{z_a^1}}{\theta_a^*} &= \ln\left(\frac{z_a^1}{z_\theta^{a,r}\left(\mathbf{x}_a^*\right)}\right) - \psi_h\left(\frac{z_a^1}{L_O^a\left(\mathbf{x}_a^*\right)}\right) \\
\frac{\kappa [\![q]\!]_0^{z_a^1}}{q_a^*} &= \ln\left(\frac{z_a^1}{z_q^{a,r}\left(\mathbf{x}_a^*\right)}\right) - \psi_h\left(\frac{z_a^1}{L_O^a\left(\mathbf{x}_a^*\right)}\right)
\end{aligned}
$$

$\longrightarrow$ Resolution via fixed-point iterations

Usual practice in coupled OA models

$$\left.\begin{array}{l} X\left(z_o^1\right) \longrightarrow X\left(0^-\right) \\ X\left(z_u^{a,r}\right) \longrightarrow X\left(0^+\right) \end{array}\right\} \longrightarrow \text{Constant profiles in AVSL and OSL}$$



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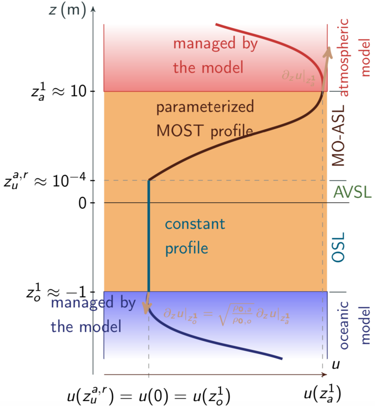

Goal: integrate these layers into the parameterizations:

- regularity
- symmetry
- justification of $X\left(z_o^1\right) \longrightarrow X\left(0^-\right)$ and $X\left(z_u^{a,r}\right) \longrightarrow X\left(0^+\right)$

(Charles Pelletier, PhD thesis 2018)

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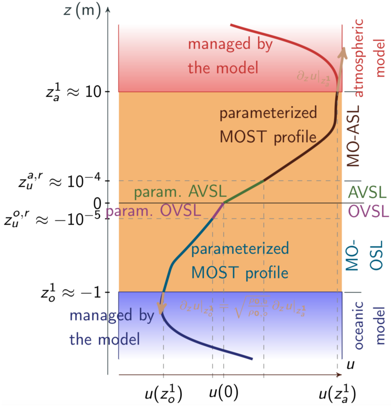

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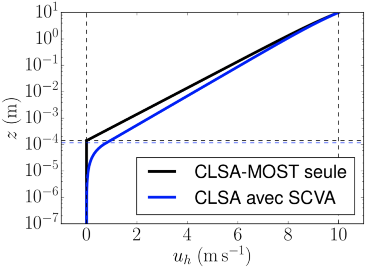
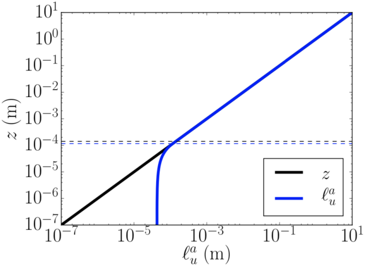
A full parameterization of OA surface layers

AVSL Atmospheric Viscous Sub-Layer

- introduction of a new viscous coordinate: $z \mapsto \ell_x^a(z)$
- unified formalism with Monin-Obukhov theory via $z \mapsto \ell_x^a(z)$
- mathematical regularity + physical parameterization





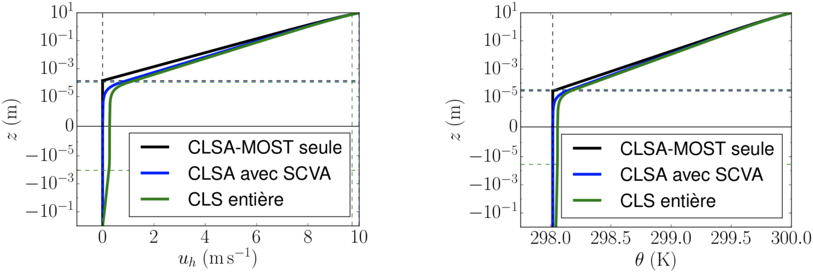
Profiles of u and ℓ_u^a with AVSL parameterization Kudryatsev (1974).

$[\![u]\!]_0^{z_a^1} = 10\ m.s^{-1}$ *and* $[\![\theta]\!]_0^{z_a^1} = +2\ K$.

A full parameterization of OA surface layers

OSL Oceanic Surface Layer

- symmetry with ASL (Monin-Obukhov + oceanic viscous coordinate $\ell_x^o(z)$)
- $\mathbf{x}_a^* \to \mathbf{x}_o^*$ via equality of turbulent fluxes: $\rho_a \nu_a^t \partial_z \mathbf{u_h}\big|_{z_a^1} = \rho_o \nu_o^t \partial_z \mathbf{u_h}\big|_{z_o^1}$
- param. OVSL $\Longleftrightarrow$ AVSL



Profiles of u and θ

$[\![u]\!]_{z_o^1}^{z_a^1} = 10\ m.s^{-1}$ and $[\![\theta]\!]_{z_o^1}^{z_a^1} = +2\ K$

Impact on closure equations

$$\frac{\kappa \left\| [\![\mathbf{u_h}]\!]_{z_o^1}^{z_a^1} \right\|}{u_a^*} = \ln\left(\frac{z_a^1}{z_u^{a,r}(\mathbf{x}_a^*)}\right) - \psi_m\left(\frac{z_a^1}{L_O^a(\mathbf{x}_a^*)}\right) + 1 \qquad \text{AVSL}$$

$$+\lambda_u \qquad \text{OVSL} \qquad \lambda_u = \sqrt{\rho_{0,a}/\rho_{0,o}}$$

$$+\lambda_u \ln\left(\frac{z_o^1}{z_u^{o,r}(\mathbf{x}_a^*)}\right) + \lambda_u \psi_m\left(\frac{-z_o^1}{L_O^o(\mathbf{x}_o^*)}\right) \qquad \text{OSL}$$

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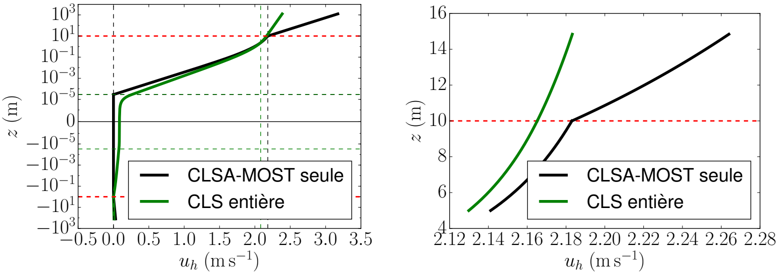

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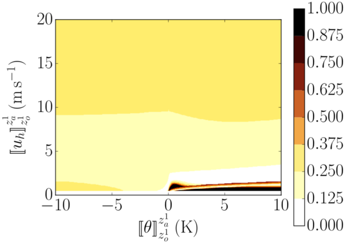
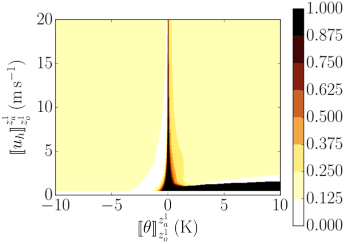


Relative difference on $|\boldsymbol{\tau}|$



Relative difference on Q_H

Impact on closure equations

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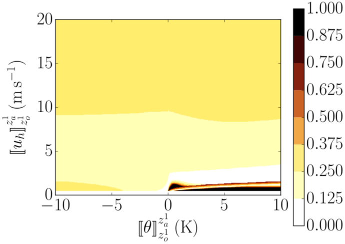
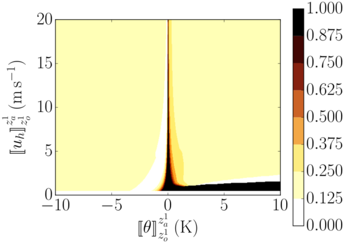
$$+\lambda_u \ln\left(\frac{z_o^1}{z_u^{o,r}(\mathbf{x}_a^*)}\right) + \lambda_u \psi_m\left(\frac{-z_o^1}{L_O^o(\mathbf{x}_o^*)}\right) \qquad \text{OSL}$$



Relative difference on $|\boldsymbol{\tau}|$



Relative difference on Q_H

⟶ Impact in realistic applications currently being tested

Pelletier C., F. Lemarié, E. Blayo, J.-L. Redelsperger and P.-E. Brilouet, 2019: Comprehensive and coupled ocean-atmosphere turbulent parameterization schemes. *In preparation*.

Remark: approach under location uncertainty

PhD thesis of B. Pinier, with E. Mémin and R. Lewandovski (2019)

$$\frac{d\mathbf{X}_t}{dt} = \tilde{\mathbf{u}}(\mathbf{X}_t, t) + \boldsymbol{\sigma}(\mathbf{X}_t, t)\,\frac{d\mathbf{B}_t}{dt} \qquad \text{with} \begin{cases} \tilde{\mathbf{u}}(\mathbf{x}, t) \text{ deterministic “large scale” velocity} \\ \boldsymbol{\sigma}(\mathbf{x}, t) \text{ convolution (diffusion) kernel} \\ \mathbf{B} \text{ Brownian motion function} \end{cases}$$

- derivation of Navier-Stokes equations *under location uncertainty*
- derivation of a modified wall law expression, with a buffer layer between the viscous sublayer and the log-layer

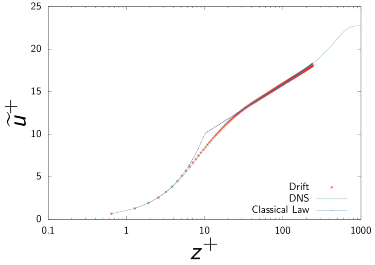


Figure: $Re_* = 600$

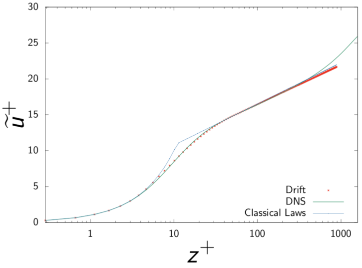
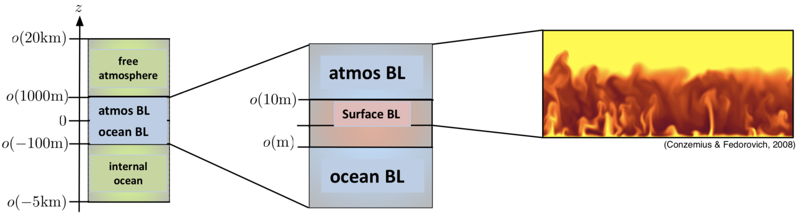


Figure: $Re_* = 2000$

Estimation of OA fluxes - In summary



$$\begin{cases} \boldsymbol{\tau} = \rho^{\mathrm{a}}\, C_D\, \|[\mathbf{u}]_o^a\|\, [\mathbf{u}]_o^a \\ Q_S = \rho^{\mathrm{a}} c_p^{\mathrm{a}}\, C_H\, \|[\mathbf{u}]_o^a\|\, [T]_o^a \end{cases}$$

$[\mathbf{u}]_o^a, [T]_o^a$ = jumps of $\boldsymbol{u}$ and T

C_D, C_H: exchange coefficients given by (complicated) bulk formulas $= f([\mathbf{u}]_o^a, [T]_o^a, [q]_o^a, z_{\mathrm{atm}}, z_{\mathrm{oce}}, \ldots)$

Keywords: Monin-Obukhov theory (wall law + stratified fluid), bulk aerodynamic formulas...

Current modeling issues

- Addition of other effects (< 1 day): gustiness, diurnal warm layers... (COCOA project)
- Well-posedness issues - coherence of the different parameterizations

$$\left\{\begin{array}{rcll} \dfrac{\partial u}{\partial t}(z,t) - \dfrac{\partial}{\partial z}\left(\nu(z, |u(z_1^a)|)\,\dfrac{\partial u}{\partial z}(z,t)\right) & = & f & z \in (z_1^a, z^\infty) \\ \nu(z_1^a, |u(z_1^a)|)\,\dfrac{\partial u}{\partial z}(z_1^a) & = & C_D\,|u(z_1^a)|\,u(z_1^a) & \\ u(z^\infty) & = & u_G & \end{array}\right.$$

- Validity issues
  - Monin-Obukhov theory: $\mathbf{u}_{ocean}(0) = 0$, rigid lid approximation
  - connection to $z = 0$: viscous sublayer
  - effect of waves !!

Algorithmic issues

We have to solve:

$$\begin{cases} \mathcal{L}_{\text{atm}}\,\boldsymbol{U}^{\text{a}} = f_{\text{atm}} & \text{in } \Omega_{\text{atm}} \times [0, T] \\ \mathcal{L}_{\text{oce}}\,\boldsymbol{U}^{\text{o}} = f_{\text{oce}} & \text{in } \Omega_{\text{oce}} \times [0, T] \\ \mathcal{F}_{\text{atm}}\,\boldsymbol{U}^{\text{a}} = \mathcal{F}_{\text{oce}}\,\boldsymbol{U}^{\text{o}} = F_{\text{OA}}(\boldsymbol{U}^{\text{a}}, \boldsymbol{U}^{\text{o}}, \mathcal{R}) & \text{on } \Gamma \times [0, T] \end{cases}$$

Algorithmic issues

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Two actual approaches:

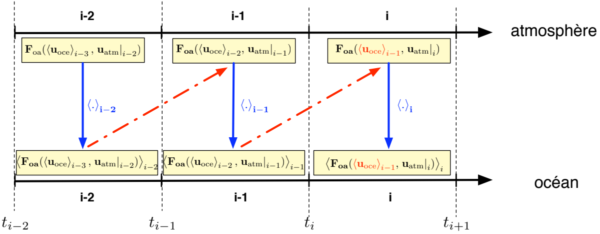
- Asynchronous coupling by time windows (averaged fluxes)
- Synchronous coupling at the time step (instantaneous fluxes)

Asynchronous coupling (time windows: $[0, T] = \cup_{i=1}^{M} [t_i, t_{i+1}]$)

$$\begin{cases} \mathcal{L}_{\text{atm}} \boldsymbol{U}^{\text{a}} = f_{\text{atm}} & \text{in } \Omega_{\text{atm}} \times [t_i, t_{i+1}] \\ \mathcal{F}_{\text{atm}} \boldsymbol{U}^{\text{a}} = F_{\text{OA}}(\langle \boldsymbol{U}^{\text{o}} \rangle_{i-1}, \boldsymbol{U}^{\text{a}}, \mathcal{R}) & \text{on } \Gamma \times [t_i, t_{i+1}] \end{cases}$$

then

$$\begin{cases} \mathcal{L}_{\text{oce}} \boldsymbol{U}^{\text{o}} = f_{\text{oce}} & \text{in } \Omega_{\text{oce}} \times [t_i, t_{i+1}] \\ \mathcal{F}_{\text{oce}} \boldsymbol{U}^{\text{o}} = \langle \mathcal{F}_{\text{atm}} \boldsymbol{U}^{\text{a}} \rangle_i & \text{on } \Gamma \times [t_i, t_{i+1}] \end{cases}$$



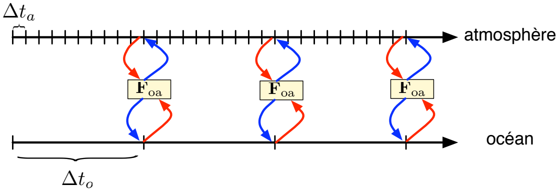
- same mean fluxes over each time window $[t_i, t_{i+1}]$
- ... but synchrony issue

Synchronous coupling at the time step

$$\begin{cases} \mathcal{L}_{\text{atm}} \boldsymbol{U}^{\text{a}} = f_{\text{atm}} & \text{in } \Omega_{\text{atm}} \times [t_i, t_i + N\,\Delta t_{\text{a}}] \\ \mathcal{F}_{\text{atm}} \boldsymbol{U}^{\text{a}} = F_{\text{OA}}(\boldsymbol{U}^{\text{o}}(t_i), \boldsymbol{U}^{\text{a}}(t), \mathcal{R}(t)) & \text{on } \Gamma \times [t_i, t_i + N\,\Delta t_{\text{a}}] \end{cases}$$

and

$$\begin{cases} \mathcal{L}_{\text{oce}} \boldsymbol{U}^{\text{o}} = f_{\text{oce}} & \text{in } \Omega_{\text{oce}} \times [t_i, t_i + \Delta t_{\text{o}}] \\ \mathcal{F}_{\text{oce}} \boldsymbol{U}^{\text{o}} = F_{\text{OA}}(\boldsymbol{U}^{\text{o}}(t_i), \boldsymbol{U}^{\text{a}}(t_i), \mathcal{R}(t_i)) & \text{on } \Gamma \times [t_i, t_i + \Delta t_{\text{o}}] \end{cases}$$

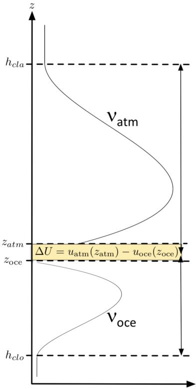


- still a (much smaller) synchrony issue
- difficult to implement efficiently
- validity issue

Stability analysis

Model problem: 1-D diffusion with variable and discontinuous coefficients

$$
\begin{aligned}
\frac{\partial u_{\text{atm}}}{\partial t} - \frac{\partial}{\partial z}\left(\nu_{\text{atm}}(z)\frac{\partial u_{\text{atm}}}{\partial z}\right) &= f_{\text{atm}} && \text{in } \Omega_{\text{atm}} \times [0,T] \\
\frac{\partial u_{\text{oce}}}{\partial t} - \frac{\partial}{\partial z}\left(\nu_{\text{oce}}(z)\frac{\partial u_{\text{oce}}}{\partial z}\right) &= f_{\text{oce}} && \text{in } \Omega_{\text{oce}} \times [0,T]
\end{aligned}
$$



with interface conditions:

Dirichlet-Neumann:
$$
\left\{
\begin{aligned}
u_{\text{atm}}(0,t) &= u_{\text{oce}}(0,t) \\
\nu_{\text{oce}}(0)\frac{\partial u_{\text{oce}}}{\partial z}(0,t) &= \nu_{\text{atm}}(0)\frac{\partial u_{\text{atm}}}{\partial z}(0,t)
\end{aligned}
\right.
$$

or

linearized bulk:

$$
\nu_{\text{atm}}(0)\frac{\partial u_{\text{atm}}}{\partial z}(0,t) = \nu_{\text{oce}}(0)\frac{\partial u_{\text{oce}}}{\partial z}(0,t) = \alpha\left(u_{\text{atm}}(0^+,t) - u_{\text{oce}}(0^-,t)\right)
$$

Stability analysis: synchronous coupling

Consider a natural discretization scheme: Euler + implicit $\longrightarrow$ each model is unconditionally stable.

Stability analysis: synchronous coupling

Consider a natural discretization scheme: Euler + implicit $\longrightarrow$ each model is unconditionally stable.

- Stability analysis shows that the coupled scheme is unstable for usual configurations of ocean-atmosphere models

$$\text{stable iff} \quad \begin{cases} \Delta z_{\text{atm}} \ll \Delta z_{\text{oce}} & \text{for Dirichlet-Neumann} \\ \alpha \ \leq \min\left(\dfrac{\nu_{\text{oce}}(0)}{\Delta z_{\text{oce}}} \dfrac{\rho_{\text{oce}}}{\rho_{\text{atm}}} \, ; \, \dfrac{\nu_{\text{atm}}(0)}{\Delta z_{\text{atm}}} \right) & \text{for linearized bulk} \end{cases}$$

Lemarié F., E. Blayo and L. Debreu, 2015: Analysis of ocean-atmosphere coupling algorithms: consistency and stability issues. *Procedia Computer Science*, **51**, 2066-2075.

- This does not mean that realistic coupled OA models systematically blow up, due to other processes (additional diffusion and viscosity).

Beljaars A., E. Dutra, G. Balsamo and F. Lemarié, 2017: On the numerical stability of surface–atmosphere coupling in weather and climate models. *Geosci. Model Dev.*, **10**, 977–989.

Conclusion on current coupling methods

- Current coupling methods are simple ad-hoc algorithms, which ensure that fluxes are balanced and are computationally cheap.
- These methods are inadequate from a mathematical point of view.

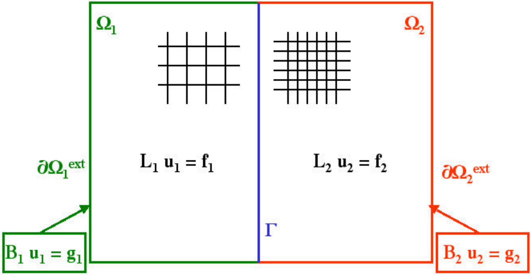
Conclusion on current coupling methods

- Current coupling methods are simple ad-hoc algorithms, which ensure that fluxes are balanced and are computationally cheap.
- These methods are inadequate from a mathematical point of view.

Issues:

- Can we improve the coupling method ?
- Does it improve the physics of the coupled solution ?
- Can this be done for a reasonable CPU cost ?

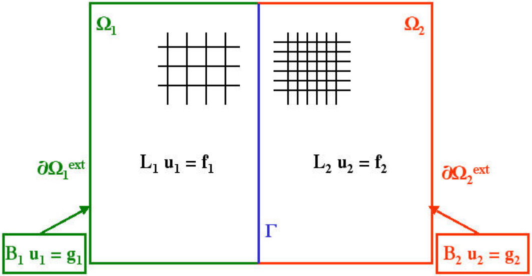
A convenient framework: Schwarz methods



$$\begin{cases} L_1 u_1 & = f_1 & \Omega_1 \times [0, T] \\ u_1 & \text{given} & \text{at } t = 0 \\ B_1 u_1 & = g_1 & \partial\Omega_1^{\text{ext}} \times [0, T] \end{cases} \qquad \begin{cases} L_2 u_2 & = f_2 & \Omega_2 \times [0, T] \\ u_2 & \text{given} & \text{at } t = 0 \\ B_2 u_2 & = g_2 & \partial\Omega_2^{\text{ext}} \times [0, T] \end{cases}$$

$+$ physical constraints at the interface : $\mathcal{F}(u_1, u_2) = 0$

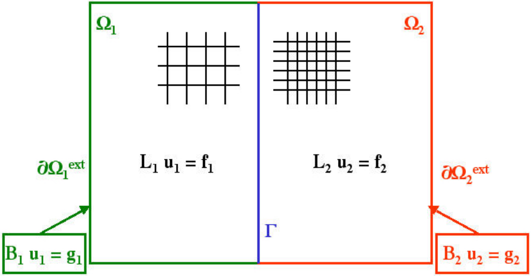
A convenient framework: Schwarz methods



$$\left\{\begin{array}{rll} L_1 u_1 & = f_1 & \Omega_1 \times [0,T] \\ u_1 & \text{given} & \text{at } t=0 \\ B_1 u_1 & = g_1 & \partial\Omega_1^{ext} \times [0,T] \\ C u_1 & = C' u_2 & \Gamma \times [0,T] \end{array}\right. \qquad \left\{\begin{array}{rll} L_2 u_2 & = f_2 & \Omega_2 \times [0,T] \\ u_2 & \text{given} & \text{at } t=0 \\ B_2 u_2 & = g_2 & \partial\Omega_2^{ext} \times [0,T] \\ D u_2 & = D' u_1 & \Gamma \times [0,T] \end{array}\right.$$

$$\text{with } (Cu_1 = C'u_2 \ \text{ and } \ Du_2 = D'u_1) \Longleftrightarrow \mathcal{F}(u_1, u_2) = 0$$

A convenient framework: Schwarz methods



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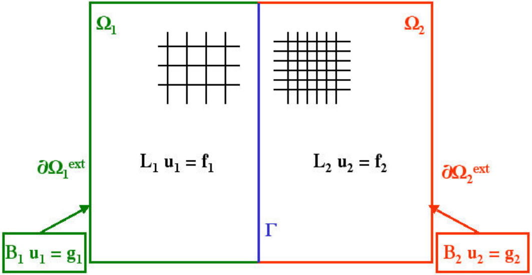

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$$\text{with } (Cu_1 = C'u_2 \ \text{ and } Du_2 = D'u_1) \Longleftrightarrow \mathcal{F}(u_1, u_2) = 0$$

- Present ocean-atmosphere coupling methods correspond to one single iteration of a Schwarz-like coupling method

Toward iterative ocean-atmosphere coupling

$$\left\{\begin{array}{lcll} \mathcal{L}_{\mathrm{atm}}\boldsymbol{U}^{\mathrm{a}} & = & f_{\mathrm{atm}} & \text{in } \Omega_{\mathrm{atm}} \times [t_i, t_{i+1}] \\ \mathcal{F}_{\mathrm{atm}}\boldsymbol{U}^{\mathrm{a}} & = & F_{\mathrm{OA}}(\boldsymbol{U}^{\mathrm{o}}, \boldsymbol{U}^{\mathrm{a}}, \mathcal{R}) & \text{on } \Gamma \times [t_i, t_{i+1}] \end{array}\right.$$

$$\left\{\begin{array}{lcll} \mathcal{L}_{\mathrm{oce}}\boldsymbol{U}^{\mathrm{o}} & = & f_{\mathrm{oce}} & \text{in } \Omega_{\mathrm{oce}} \times [t_i, t_{i+1}] \\ \mathcal{F}_{\mathrm{oce}}\boldsymbol{U}^{\mathrm{o}} & = & \mathcal{F}_{\mathrm{atm}}\boldsymbol{U}^{\mathrm{a}} & \text{on } \Gamma \times [t_i, t_{i+1}] \end{array}\right.$$

Toward iterative ocean-atmosphere coupling

Iterate until convergence

$$\begin{cases} \mathcal{L}_{\text{atm}} \boldsymbol{U}^{\text{a}}_{k+1} = f_{\text{atm}} & \text{in } \Omega_{\text{atm}} \times [t_i, t_{i+1}] \\ \mathcal{F}_{\text{atm}} \boldsymbol{U}^{\text{a}}_{k+1} = F_{\text{OA}}(\boldsymbol{U}^{\text{o}}_{k}, \boldsymbol{U}^{\text{a}}_{k+1}, \mathcal{R}_{k+1}) & \text{on } \Gamma \times [t_i, t_{i+1}] \end{cases}$$

then

$$\begin{cases} \mathcal{L}_{\text{oce}} \boldsymbol{U}^{\text{o}}_{k+1} = f_{\text{oce}} & \text{in } \Omega_{\text{oce}} \times [t_i, t_{i+1}] \\ \mathcal{F}_{\text{oce}} \boldsymbol{U}^{\text{o}}_{k+1} = \mathcal{F}_{\text{atm}} \boldsymbol{U}^{\text{a}}_{k+1} & \text{on } \Gamma \times [t_i, t_{i+1}] \end{cases}$$

Impact on the physics

A major difficulty There is no idealized coupled ocean-atmosphere testcase with a known reference solution.

Impact on the physics

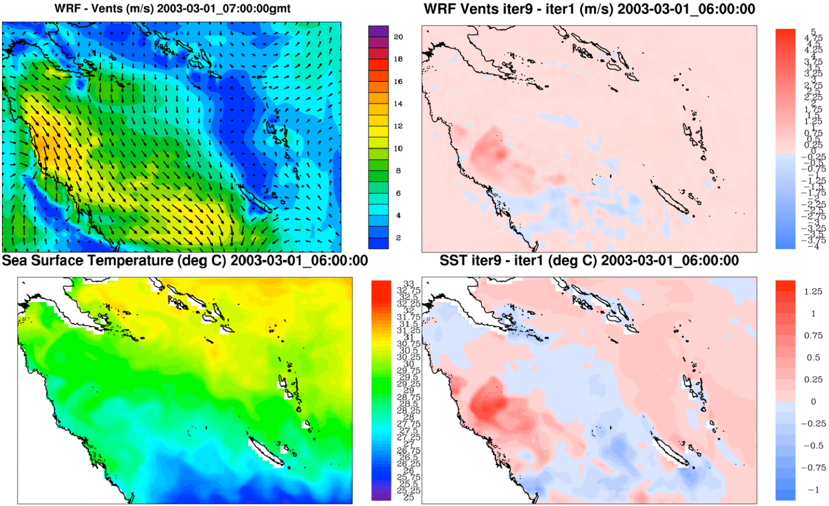
Simulation of the tropical cyclone Erica (2003), by coupling

- ROMS: primitive equation ocean model (Shchepetkin-McWilliams, 2005)
- WRF: non hydrostatic atmospheric model (Skamarock-Klemp, 2007)

$\Delta x_{\text{atm}} = 35\text{km}$, $\Delta t_{\text{atm}} = 180\text{s}$
$\Delta x_{\text{oce}} = 18\text{km}$, $\Delta t_{\text{oce}} = 1800\text{s}$
15-day simulation

Interface conditions: vertical fluxes for momentum, heat and fresh water

Impact on the physics (cont'd)



10-meter wind (m/s) and sea surface temperature (°C).

Impact on the physics (cont'd)

To assess the robustness of the coupled solution: ensemble simulations

- Initial conditions
- Length of the time windows: 6h vs 3h

Impact on the physics (cont'd)

To assess the robustness of the coupled solution: ensemble simulations

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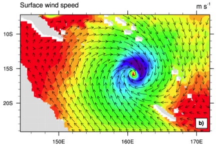
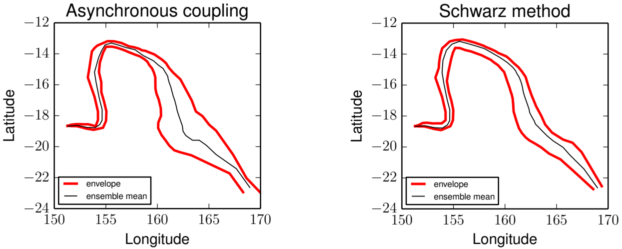
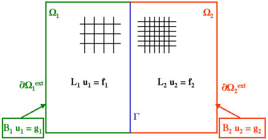


Trajectory of the cyclone



- The uncertainty on the cyclone trajectory and intensity is decreased by 30%-40%. (see Lemarié et al, 2014, for further details)

Decreasing the cost: absorbing boundary conditions



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Systems satisfied by the errors $e_i^k = u_i^k - u_i$:

$$\begin{cases} L_1 e_1^{k+1} & = 0 & \Omega_1 \times [0,T] \\ e_1^{k+1} & = 0 & \text{at } t=0 \\ B_1 e_1^{k+1} & = 0 & \partial\Omega_1^{\text{ext}} \times [0,T] \\ C e_1^{k+1} & = C' e_2^k & \Gamma \times [0,T] \end{cases} \qquad \begin{cases} L_2 e_2^{k+1} & = 0 & \Omega_2 \times [0,T] \\ e_2^{k+1} & = 0 & \text{at } t=0 \\ B_2 e_2^{k+1} & = 0 & \partial\Omega_2^{\text{ext}} \times [0,T] \\ D e_2^{k+1} & = D' e_1^k & \Gamma \times [0,T] \end{cases}$$

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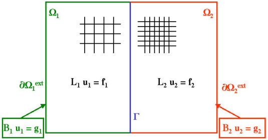

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If one finds C', D' such that $C' e_2 = 0$ and/or $D' e_1 = 0$, then convergence in 2 iterations. $\longrightarrow$ **exact absorbing conditions** (Engquist & Majda, 1977)

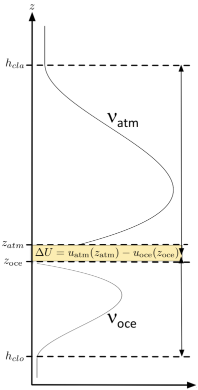
Improving the convergence speed

Model problem: coupling of two Ekman layers

$$
\begin{aligned}
\partial_t u_{\text{atm}} - f\, v_{\text{atm}} - \partial_z \left(\nu_{\text{atm}}(z) \partial_z u_{\text{atm}} \right) &= F^x_{\text{atm}} \\
\partial_t v_{\text{atm}} + f\, u_{\text{atm}} - \partial_z \left(\nu_{\text{atm}}(z) \partial_z v_{\text{atm}} \right) &= F^y_{\text{atm}} \qquad \text{in } \Omega_{\text{atm}} \times [0, T] \\
u_{\text{atm}}(z, t = 0) &= u_0(z) \qquad z \in \Omega_{\text{atm}}
\end{aligned}
$$

$$
\begin{aligned}
\partial_t u_{\text{oce}} - f\, v_{\text{oce}} - \partial_z \left(\nu_{\text{oce}}(z) \partial_z u_{\text{oce}} \right) &= F^x_{\text{oce}} \\
\partial_t v_{\text{oce}} + f\, u_{\text{oce}} - \partial_z \left(\nu_{\text{oce}}(z) \partial_z v_{\text{oce}} \right) &= F^y_{\text{oce}} \qquad \text{in } \Omega_{\text{oce}} \times [0, T] \\
u_{\text{oce}}(z, t = 0) &= u_0(z) \qquad z \in \Omega_{\text{oce}}
\end{aligned}
$$

$$
\begin{pmatrix} u_{\text{atm}} \\ v_{\text{atm}} \end{pmatrix} = \begin{pmatrix} u_{\text{oce}} \\ v_{\text{oce}} \end{pmatrix} \text{ and } \nu_{\text{atm}}(0)\, \partial_z \begin{pmatrix} u_{\text{atm}} \\ v_{\text{atm}} \end{pmatrix} = \nu_{\text{oce}}(0) \partial_z \begin{pmatrix} u_{\text{oce}} \\ v_{\text{oce}} \end{pmatrix} \quad \text{on } \Gamma \times [0, T]
$$



Difficulties:

- coupling of u and v
- variable-in-space diffusivity + discontinuity at $z = 0$

Improving the convergence speed

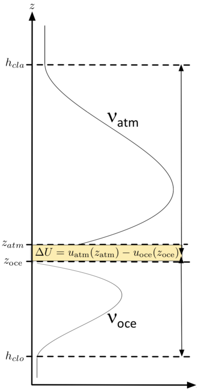
Model problem: coupling of two Ekman layers

Schwarz iteration:

$$
\begin{array}{rcll}
\partial_t u_{\text{atm}}^k - f\, v_{\text{atm}}^k - \partial_z \left(\nu_{\text{atm}}(z)\partial_z u_{\text{atm}}^k\right) & = & F_{\text{atm}}^x & \\
\partial_t v_{\text{atm}}^k + f\, u_{\text{atm}}^k - \partial_z \left(\nu_{\text{atm}}(z)\partial_z v_{\text{atm}}^k\right) & = & F_{\text{atm}}^y & \text{in } \Omega_{\text{atm}} \times \\
u_{\text{atm}}^k(z, t=0) & = & u_0(z) & z \in \Omega_{\text{atm}} \\
\mathcal{C}\, u_{\text{atm}}^k(0,t) & = & \mathcal{C}'\, u_{\text{atm}}^{k-1}(0,t) & t \in [0,T]
\end{array}
$$

$$
\begin{array}{rcll}
\partial_t u_{\text{oce}}^k - f\, v_{\text{oce}}^k - \partial_z \left(\nu_{\text{oce}}(z)\partial_z u_{\text{oce}}^k\right) & = & F_{\text{oce}}^x & \\
\partial_t v_{\text{oce}}^k + f\, u_{\text{oce}}^k - \partial_z \left(\nu_{\text{oce}}(z)\partial_z v_{\text{oce}}^k\right) & = & F_{\text{oce}}^y & \text{in } \Omega_{\text{oce}} \times [0, \\
u_{\text{oce}}^k(z, t=0) & = & u_0(z) & z \in \Omega_{\text{oce}} \\
\mathcal{D}\, u_{\text{oce}}^k(0,t) & = & \mathcal{D}'\, u_{\text{oce}}^{k}(0,t) & t \in [0,T]
\end{array}
$$

for a given $(u_{\text{oce}}^0, v_{\text{oce}}^0)$ on Γ.



Improving the convergence speed (cont'd)

		Constant diffusion	Space dependent diffusion
Steady state	No Coriolis effect	**Dubois (2007)** opt. Schwarz 2-D adv-diff eq. **Gander-Zhang (2016)** opt. Schwarz Helmholtz eq.	**Lions (1990)** convergence Schwarz diffusion eq.
	Coriolis effect		
Time dependent	No Coriolis effect	**Gander-Halpern (2002)** opt. Schwarz heat eq. **Blayo-Rousseau-Tayachi (2017)** lin. viscous SW eq. **Bennequin-Gander-Gouarin-Halpern (2004)** opt. Schwarz 2-D diff-reaction	**Lemarié-Debreu-Blayo (2013)** opt. Schwarz 1-D diffusion
	Coriolis effect	**Martin (2003)** opt. Schwarz 2-D SW **Audusse-Dreyfuss-Merlet (2010)** opt. Schwarz 3-D primitive eqs	

Improving the convergence speed (cont'd)

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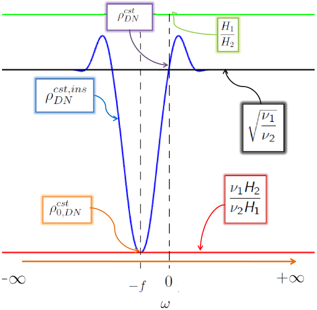
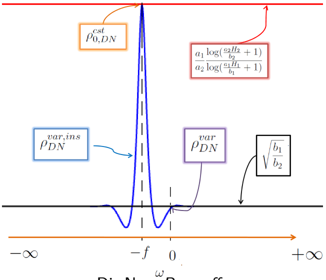
Improving the convergence speed (cont'd)

Exact analytical expression for the convergence factor:

- P_0, P_1 and P_2 diffusion profiles
- Dirichlet-Neumann and Robin-Robin interface conditions
- optimized coefficients for Robin-Robin conditions



Dir-Neu, cst coeff



Dir-Neu, P_1 coeff

Blayo E., F. Lemarié, C. Pelletier and S. Thery, 2019: Coupling two Ekman layers with a Schwarz algorithm. *In preparation*.

Current and future work on coupling algorithms

- Integrate all these results in a model of the three boundary layers (atmospheric, surface, oceanic)

In collaboration with climate scientists:

- Build 1D coupled reference test cases (idealized and realistic)
- Include this coupling strategy in the French climate models
- Mitigate the cost (perform the iterations only for the boundary layers, model reduction...)